Unbiased optimal power flow for power systems with wind power generation

S.J. Plathottam, P. Ranganathan and H. Salehfar

Wind power plants connected to power systems are often unable to utilise all available power due to transmission constraints. This problem is illustrated in an optimal power flow problem to schedule power from two different wind plants in a 6-bus system. A cost function is proposed to schedule wind power in an unbiased manner.

Introduction: Optimal power flow (OPF) is an enhanced form of the economic dispatch (ED) optimisation problem where both active and reactive powers are treated as control variables and adjusted to minimise generation cost while observing associated power flow constraints. The constraints include but are not limited to thresholds on bus voltages, reactive power generation and the thermal ratings of transmission lines. The large-scale penetration of wind generation has created major technical challenges for power system operation which in turn manifests itself in the associated ED and OPF problems for the concerned power system [1]. This Letter discusses an OPF problem for a power system consisting of thermal and wind turbine generators (WTGs) and proposes a cost function to promote equitable scheduling of power from multiple wind power plants.

Motivation: One of the key challenges faced during integration of wind farms with the grid is the spillage due to transmission constraints. A case study performed in [2] has shown that as much as 30% of available wind power may be spilled during periods of high wind power availability without the use of reactive power compensation technologies like flexible AC transmission systems. Although spillage can be reduced by upgrading the grid infrastructure or by introducing energy storage, economic and societal factors often impede such solutions. Admittance properties of the power system may also cause more spillage in certain plants compared with others. Hence a more nuanced approach is required for providing an unbiased treatment to all power plant operators.

OPF with wind generation: The standard OPF problem has been discussed in detail in [3]. The optimisation problem can be stated as determining the day-ahead dispatch for a system consisting of m buses, n_thermal thermal plants and n_w wind plants. The objective is to minimise the cost given by (1) which is the summation of the costs due to scheduled wind power generation (P_{wind,i,j}), thermal power generation (P_{therm,j,i}) and transmission loss (P_{loss,i}) over N time blocks. The length of each block is 24/N hours. Thermal generation cost is given by a polynomial cost function (C_{therm}), whereas P_{loss,i} is the cost per unit of energy lost in transmission. Equation (2) is the overall active power balance constraint, whereas the AC power flow equations are given by (3) and (4). Here admittances between the 8th and 7th buses (G_{87} + jB_{87}) are taken in rectangular coordinates, whereas voltage at the 8th bus (V_{8j}δ_{8j}) is taken in polar coordinates. Physically they convey

\[
\min \sum_{i=1}^{N} \sum_{j=1}^{n} C_i(P_{therm,j,i}) + \sum_{i=1}^{N} \sum_{j=1}^{n} C_w(P_{wind,j,i}) + R_{whole} \sum_{i=1}^{N} P_{loss,i} 
\]

(1)

\[
P_{gen,k,i} - P_{load,k,i} = \sum_{j=1}^{m} V_{kj}^i (G_{kj,i} \cos(\delta_{k,i} - \delta_{j,i}) + B_{kj,i} \sin(\delta_{k,i} - \delta_{j,i}) )
\]

(2)

\[
Q_{gen,k,i} - Q_{load,k,i} = \sum_{j=1}^{m} V_{kj}^i (G_{kj,i} \sin(\delta_{k,i} - \delta_{j,i}) + B_{kj,i} \cos(\delta_{k,i} - \delta_{j,i}) )
\]

(3)

that the active (P_{gen,k,i}) and reactive (Q_{gen,k,i}) powers generated in the kth bus minus the load at that bus (P_{load,k,i}, Q_{load,k,i}) is equal to the net power flowing out of the bus. Active and reactive powers for thermal generation are constrained by the generator ratings shown in Fig. 1. The reactive power limit for wind generation is calculated based on a power factor of 0.95 lag/lead [4]. Voltage limits for all buses are set between 0.9 and 1.1 p.u. It may be pointed out that generation ramp rate limits and transmission line capacity constraints have not been taken into consideration.

Cost function for unscheduled wind: Since there is no fuel cost associated with wind energy, power scheduled from a wind plant cannot be assigned a cost in the traditional sense. It has been pointed out in [5] that at high levels of wind capacity penetration, system operational costs can increase because of the variable nature of wind. The levelised cost of energy (LCOE) which is calculated using a net present value analysis is an indicator of the generation cost of wind energy [6] and is calculated using (5). LCOE was used as a performance index in the coordinated output control of multiple DG resources in [7]

\[
\frac{C_{\text{LCOE}}}{8760} = \frac{C_{\text{wind}}}{8760} \left( \frac{1}{P_{\text{rated wind}} CF_{\text{design}}^{\text{wind}}} \right) + \left[ 1 + M \left( \frac{1}{D(1 + \frac{D}{D})} \right) \right] \]

(5)

From (5) it can be seen that for a plant of rated capacity (P_{\text{rated wind}}) with fixed investment parameters like the amount of capital invested (C_{\text{wind}}), discount rate (\Omega), lifetime of plant (n) and annual maintenance cost fraction (\alpha), the design capacity factor (CF_{\text{design}}^{\text{wind}}) of the wind plant heavily influences LCOE. To enable the wind plant to be financially viable enough power must be scheduled so that it can maintain an average capacity factor (CF_{\text{average}}^{\text{wind}}) that is close to the design capacity factor. This Letter proposes calculating the opportunity cost for unscheduled wind power (C_{\text{opportunity}}^{\text{wind}}) using (6) based on the difference between the actual and design LCOE (\Delta C_{\text{LCOE}}^{\text{wind}}) given by (7). The authors assume that the short-term estimated wind power (P_{\text{predicted wind}}) is available to the OPF problem from physical and/or statistical models. A good overview on short-term wind power prediction methodologies is presented in [8]. Additionally, to quantify the amount of predicted wind utilised by wind plants a wind utilisation factor (U_{\text{wind}}) given by (8) is defined

\[
C_{\text{opportunity}}^{\text{wind}} = \Delta C_{\text{LCOE}}^{\text{wind}} = \left( \frac{1}{P_{\text{rated wind}} CF_{\text{design}}^{\text{wind}}} \right) \left( \frac{C_{\text{wind}}}{8760} \right) \left[ 1 + M \left( \frac{1}{D(1 + \frac{D}{D})} \right) \right] \]

(6)

\[
U_{\text{wind}} = \frac{\sum_{i=1}^{N} P_{\text{wind},i}}{\sum_{i=1}^{N} P_{\text{rated wind}}}
\]

(7)

Case studies and results: The 6-bus system shown in Fig. 1 is used in this Letter. The bus admittance matrix is constructed using the data from [9] where each line has an impedance of 0.04 + 0.08j. The load profile shown in Fig. 2 is used and the reactive load is calculated by assuming a lagging power factor of 0.85 for all loads.
In the first scenario we solve the OPF problem to minimise only the thermal plant generation cost and transmission loss, i.e. the wind power cost function is not included. However, the power generation is scheduled for \( N \) time blocks for all the plants. The problem is solved for three different predicted wind power profiles and the results are tabulated in Table 1. It can be seen that the OPF solution consistently allows the wind plant connected to bus 3 to utilise more of its predicted power compared with that connected to bus 2. Another correlation that can be noted is that a higher wind power availability (predicted) does not translate to a higher wind utilisation.

Table 1: OPF solution for first scenario

<table>
<thead>
<tr>
<th>Wind power profile</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average predicted wind power (MW)</td>
<td>159.2</td>
<td>136.7</td>
<td>113.3</td>
</tr>
<tr>
<td>Wind utilisation WTG1 (%)</td>
<td>70.7</td>
<td>72.4</td>
<td>86.1</td>
</tr>
<tr>
<td>Wind utilisation WTG2 (%)</td>
<td>97.6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total wind contribution (%)</td>
<td>61.3</td>
<td>53.9</td>
<td>48.2</td>
</tr>
<tr>
<td>Energy loss (MWh)</td>
<td>294</td>
<td>287.5</td>
<td>303.7</td>
</tr>
<tr>
<td>Load bus 1 minimum voltage (p.u.)</td>
<td>1.01</td>
<td>1.02</td>
<td>1.01</td>
</tr>
<tr>
<td>Load bus 2 minimum voltage (p.u.)</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>WTG1 minimum voltage (p.u.)</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>WTG2 minimum voltage (p.u.)</td>
<td>1.01</td>
<td>1.03</td>
<td>1.02</td>
</tr>
</tbody>
</table>

In the second scenario we include the cost function defined by (6) and (7). We assume that \( C_{LCOE}^{\text{w}} \) of wind plants at bus 2 and bus 3 are 25 and 35\% respectively. This amounts to a \( \Delta C_{\text{LCOE}}^{\text{wind}} \) of 10.3 and 2.45 $/MWh, for the two wind plants. The load and wind profiles will remain the same as in the previous scenario. The results are tabulated in Table 2.

Table 2: OPF solution with wind power cost function

<table>
<thead>
<tr>
<th>Wind power profile</th>
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<td>48.2</td>
</tr>
<tr>
<td>Energy loss (MWh)</td>
<td>369</td>
<td>317</td>
<td>303.7</td>
</tr>
<tr>
<td>Load bus 1 minimum voltage (p.u.)</td>
<td>1.01</td>
<td>0.98</td>
<td>1.01</td>
</tr>
<tr>
<td>Load bus 2 minimum voltage (p.u.)</td>
<td>0.97</td>
<td>0.92</td>
<td>0.97</td>
</tr>
<tr>
<td>WTG1 minimum voltage (p.u.)</td>
<td>1.1</td>
<td>1.05</td>
<td>1.11</td>
</tr>
<tr>
<td>WTG2 minimum voltage (p.u.)</td>
<td>1.01</td>
<td>1.02</td>
<td>1.01</td>
</tr>
<tr>
<td>Thermal plant 1 CF (%)</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Thermal plant 2 CF (%)</td>
<td>37.8</td>
<td>50.6</td>
<td>60.6</td>
</tr>
</tbody>
</table>

It can be observed that the OPF solution is scheduling more power for the wind plant at bus 2 in all cases. Interestingly, the contribution of wind energy towards meeting the total energy demand remains nearly the same irrespective of the cost functions. However, the losses are comparatively much higher with the wind cost function. It is also observed that thermal plants are scheduled in such a way that the most efficient plant has the higher capacity factor.

The wind power scheduled for each time block in scenario two using wind profile 1 is shown in Fig. 2. The voltage profile at the load buses generated by the OPF solution with and without wind power cost is shown in Fig. 3. It can be observed that the load voltages are slightly less in certain time blocks for the OPF solution using the wind power cost function compared with that without the same.

**Fig. 2** Comparison of power scheduled from both wind power plants

Further discussion: To point out the merit of the proposed cost function for wind power, the cost function proposed in [10] is reproduced in its original form as shown in (9). This function penalises the unscheduled wind power production and was part of a comprehensive cost function in an ED problem

\[
C_{\text{p,w}}(W_{\text{c},w} - w_i) = k_{\text{p,w}}(W_{\text{c},w} - w_i) \tag{9}
\]

In (9) the term \( W_{\text{c},w} - w_i \) is the difference between the available and scheduled wind power, whereas \( k_{\text{p,w}} \) is the penalty cost coefficient. It can be observed that (9) is identical to (6) except for a key difference. The term \( k_{\text{p,w}} \), which was an arbitrary value in [10] has been replaced by \( \Delta C_{\text{LCOE}}^{\text{wind}} \), which is calculated based on the actual capacity factors of the wind plants.

**Conclusion:** An OPF optimisation problem for power systems with thermal and wind power plants as been investigated. A formulation to calculate the opportunity cost of unscheduled predicted wind power production is proposed and compared with the existing literature. Results from case studies on a 6-bus system are presented. This work may be extended to systems with a larger number of buses and additional constraints.

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One or more of the Figures in this Letter are available in colour online.
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