Wind Turbine: Angle of Attack

With the rising cost of fossil fuels, both monetarily and ecologically, the use of wind turbines, such as the one shown in Fig. 1, are becoming more common. One of the major challenges with the construction of wind turbines is maximizing the return on investment—efficiently converting wind to electricity.

To achieve more efficient energy production research has been done to identify the most efficient blade profile. The issue with any profile is that its performance is a function of wind speed. A profile that maximizes performance at low wind speeds may perform poorly with increased wind speed. To minimize losses in performance the angle of attack of the blade, depicted as $\theta$ in Fig. 2, is controlled by rotating the blade about its axis. The amount the blade is rotated about its axis is referred to as the pitch angle.

The S809 wind foil, a commonly used turbine blade profile, has been tested in a wind tunnel with wind speeds of 7.2, 8, 9, and 10.5 m/s. At each wind speed the relationship between rotor torque and blade pitch was evaluated. The pitches evaluated are shown with black marks in Fig. 3. The lines that pass through the points represent polynomials fit to the data. The equation for the curves fit to the data for the wind speeds of 10.5 m/s and 7.2 m/s are provided on the plot. These equations relate pitch angle, $x$, to the rotor torque, $y$. The optimal pitch is defined as the angle that maximizes the rotor torque with a given wind speed. This pitch corresponds to the point where the derivative of the polynomial fit to the data is zero.

1. Please identify the optimal blade pitch—the pitch that produces the maximum torque—for a wind speed of 10.5 m/s. To accomplish this, take the derivative of the polynomial fit to the data. Set the resulting polynomial equal to zero. The solution to the equation can be found using the quadratic formula (See your textbook).
2. Repeat for a wind speed of 7.2 m/s.
3. Plot both the points on the graph in Fig. 3 and draw a line through the points. Does the line intersect the curves for wind speeds of 9 m/s and 8 m/s at locations corresponding to their optimal pitches?